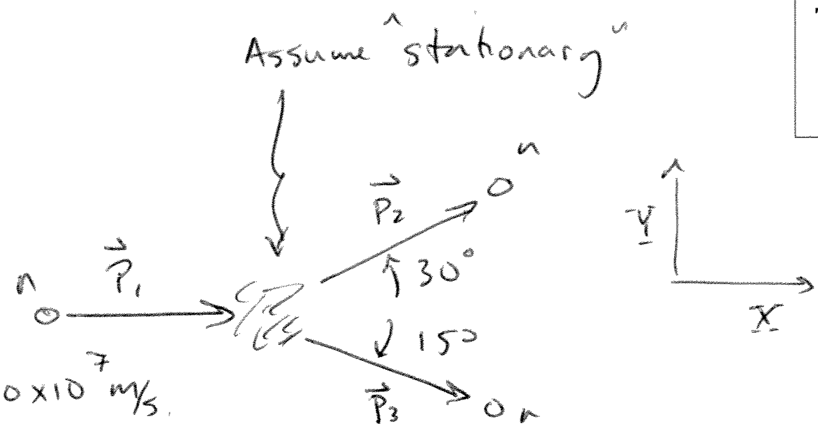
General Problem Solving Guide

Date: Nov 19 2012
Recorder: Fermi
Skeptic: Noether
Timekeeper: Einstein
Psychic: Hisenberg

20

List given information, define variables, sketch picture:

4



$V_1 = 2.0 \times 10^7$ m/s

$m_N = 1.674 \times 10^{-27}$ kg
$1 eV = 1.602 \times 10^{-19}$ J

Neutron interacts with nucleus knocking out

Simplify question, list target quantity:

1

Find energy lost (kinetic energy) which will be the internal energy increase.

List all related quantitative relationships:

$KE = \frac{1}{2}mv^2 \qquad \vec{p} = m\vec{v}$

3

$\Delta \vec{P}_{sys} = 0$ momentum is conserved in collision

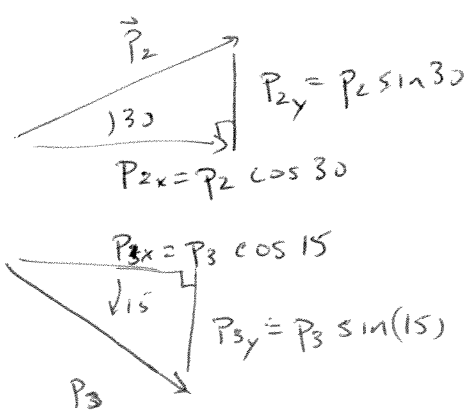
$\vec{P}_i = \vec{P}_f$ X: $P_{ix} = P_{fx}$
Y: $P_{iy} = P_{fy}$

Outline approach, sketch diagrams if needed (or sketch next to pictures above):

2

$P_{1x} = P_1$

$P_{1y} = 0$



(1) Break into components and solve for P_2 and P_3 (esentially V_2 and V_3)

(2) Use $\Delta KE = E_{internal}$ to find increase in internal energy.

Obtain a general solution:

Momentum conservation

Y: $0 = p_2 \sin(30) - p_3 \sin(15)$

X: $p_1 = p_2 \cos(30) + p_3 \cos(15)$

$p = mv \quad m_1 = m_2 = m_3 = m_N$

$\therefore$

$$0 = v_2 \sin 30 + v_3 \sin 15$$

$$v_2 = v_3 \frac{\sin 15}{\sin 30}$$

$\boxed{2}$

$$v_1 = v_2 \cos 30 + v_3 \cos 15$$

$$= v_3 \frac{\cos 30 \sin 15}{\sin 30} + v_3 \cos 15$$

$$v_1 = v_3 \left(\cos 15 + \frac{\cos 30 \sin 15}{\sin 30} \right)$$

$$= v_3 \cos(15) \left(1 + \frac{\tan 15}{\tan 30} \right)$$

$$\frac{v_1}{\cos(15)\left[1 + \frac{\tan 15}{\tan 30}\right]} = v_3 = 1.41 \times 10^7 \, m/s$$

$$v_2 = v_3 \frac{\sin(15)}{\sin 30} = \frac{v_1 \tan(15)}{\sin 30 \left[1 + \frac{\tan 15}{\tan 30}\right]}$$

$$= \frac{v_1 \tan 15}{\sin 30 \frac{\tan 15}{\tan 30} \left[\frac{\tan 30}{\tan 15} + 1\right]}$$

$$v_2 = \frac{v_1}{\cos(30)\left[1 + \frac{\tan 30}{\tan 15}\right]} = 0.732 \times 10^7 \, m/s$$

$$KE_1 = \frac{1}{2} m v_1^2 = \frac{1}{2}(1.674 \times 10^{-27} \, kg)(2.0 \times 10^7 \, \tfrac{m}{s})^2 = 3.348 \times 10^{-13} \, J$$

$$KE_2 = \frac{1}{2} m v_2^2$$

$$KE_3 = \frac{1}{2} m v_3^2$$

Check Units:

$\frac{m}{s} = \frac{m}{s}$ trig functions are unitless ✓

$J = kg \frac{m^2}{s^2} = kg \frac{m}{s^2} \cdot m = Nm$ ✓ $\boxed{2}$

Check Limiting Cases:

angles same ... $v_2 = v_3$ (equations symmetric) ✓ $\boxed{2}$

$[\theta_2 + \theta_3 = 90° \quad \Delta KE = 0]$ Advanced Hypothesis

$\theta_i = 0$ impossible ✓ (either angle must not be $\emptyset$)

$\rightarrow \theta_2 = \theta_3 = 45° \quad v_2 = v_3 = 1.41 \times 10^7 \, m/s = \frac{1}{\sqrt{2}} v_1$

$KE_1^2 = KE_2^2 + KE_3^2$ ✓

Obtain a numeric solution:

(i.e. plug in the numbers)

$|v_2| = 7.32 \times 10^6 \, m/s \quad KE_2 = 4.485 \times 10^{-14} \, J$

$v_3 = 1.41 \times 10^7 \, m/s \quad KE_3 = 1.674 \times 10^{-13} \, J$ $\boxed{1}$

$KE_1 - (KE_2 + KE_3) = \Delta KE = 1.226 \times 10^{-13} \, J$

$= 0.765 \, MeV$

Why is solution reasonable? Explain.

- Neutron at larger angle is slower
- units work
- limiting cases make sense

$\boxed{3}$